Problem 3
Beacons

Input File: *standard input*
Output File: *standard output*

Time and Memory Limits: 1 second, 256 MB

With the help of the eccentric Dr. Yes, you managed to save some of your country from the outbreak of *The Virus*[2]. Unfortunately, *The Virus* spread to other countries before you were able to deploy Dr. Yes's vaccine worldwide. Now Dr. Yes is convinced that the only hope for humanity is to attract the attention of extraterrestrial life.

To do so, Dr. Yes has built $2^N - 1$ special beacons in his (flat and very large) backyard. The beacons are indexed from 1 to $2^N - 1$, with the i-th beacon at coordinates (x_i, y_i). No two beacons lie at the same point, and no three beacons lie on a straight line. All that remains now is to activate them by connecting them.

To do this, Dr. Yes needs you to choose some pairs of beacons and join them with straight lengths of wire. To attract the attention of extraterrestials, you must also demonstrate that humans are intelligent, and Dr. Yes has concluded that you must connect the beacons so that a *complete binary tree* is formed with the beacons as nodes and the wires as edges.
Specifically, you must place $2^N - 2$ straight lengths of wire that join beacons such that:

- No two wires intersect at any point except beacons.
- Exactly one beacon has two other beacons connected to it by wire, this is the "root" beacon.
- Exactly 2^{N-1} beacons have only one other beacon connected to it, and their shortest path to the root beacon has precisely $N - 1$ pieces of wire in it. These beacons are the "leaf" beacons.
- All other $2^{N-1} - 2$ non-root, non-leaf beacons must be connected to precisely 3 other beacons.
- The total length of wire used is minimised.

The length of a wire between two beacons at points (x_i, y_i) and (x_j, y_j) is $\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ (i.e. the Euclidean distance between the two beacons). The total length of wire used is the sum of the lengths of all wires between the chosen pairs of beacons.

You are not required to find the best possible solution. Instead you must simply use the smallest total length of wire you can. Your solution will be compared against the judges' solutions, and better solutions will score more points. See the *Scoring* section for details.

Input

- The first line of input will contain a single integer N, with the number of beacons being $2^N - 1$.
- The following $2^N - 1$ lines of input will each contain two space-separated integers. The i-th of these lines will contain x_i y_i, the location of the beacon with index i.

[2]See FARIO issue #2011, 'The Virus'

Output

Your program must output $2^N - 2$ lines, each containing two integers a_i b_i, indicating that there is a straight length of wire between beacon a_i and beacon b_i. These lines may be in any order, and the ordering of the integers on each line does not matter.

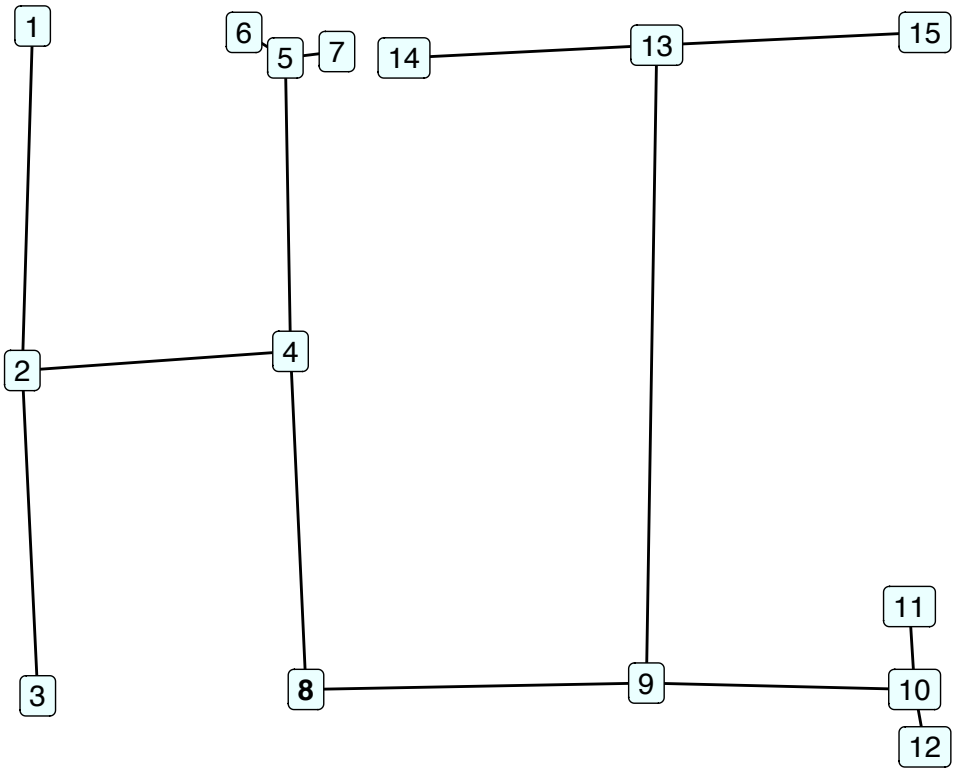
Sample Input

```
4
2 0
0 54
3 105
52 51
51 5
43 1
61 4
55 104
121 103
173 104
172 91
175 113
123 3
74 5
175 1
```

Sample Output

```
8 4
8 9
4 2
4 5
9 13
9 10
2 3
2 1
5 6
5 7
13 14
13 15
10 11
10 12
```

Explanation



We can see that beacon 8 is the root node of the tree, since it is the only node with two edges connected to it. From there, it has children 4 and 9 on its next level. 4 has children 2 and 5, then

2 has children 3 and 1, which have no children of their own. Thus 1 and 3 are 3 levels below the root node 8, and all non-leaf nodes have two children. Similarly all other leaf nodes (6, 7, 14, 15, 11, 12) are 3 levels below the root. This means that the tree we've described is a complete binary tree. Further, by observation it is a valid tree since none of the wires intersect.

Summing up the Euclidean lengths of the wires in the tree, we find that we have used 616.6759 metres of wire.

Scoring

For each test case, your solution will obtain a score determined as follows:

- If your solution does not describe a complete binary tree with no self-intersections, it will obtain a score of 0%.
- Otherwise, let P be the average distance between two beacons multiplied by $2^N - 2$, and let Q be the smallest total length of wire used by the judges' solutions. Your solution will be scored on a linear scale with P scoring 0% and Q scoring 90%, rounded up to at least 30%, and down to at most 100%. In other words your score will be $min(100, max(30, 90 \times \frac{P-X}{P-Q}))$, where X is the total length of wire used by your solution. You are guaranteed that $P > Q$.

For each subtask, your solution will receive the minimum score it obtained for any case in the subtask.

Subtasks & Constraints

For all subtasks, $0 \leq x_i, y_i \leq 1\,000\,000\,000$.

- For Subtask 1 (15 points), $N = 2$
- For Subtask 2 (15 points), $N = 3$
- For Subtask 3 (10 points), $N = 4$
- For Subtask 4 (10 points), $N = 5$
- For Subtask 5 (10 points), $N = 6$
- For Subtask 6 (10 points), $N = 7$
- For Subtask 7 (10 points), $N = 8$
- For Subtask 8 (10 points), $N = 9$
- For Subtask 9 (10 points), $N = 10$